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COGNITIVE DYNAMICS ON TOPOLOGICAL DOMAINS

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ABSTRACT

Analysis of psychological and behavioral dynamics via mathematics has been a major focus among Psychologists, Psychiatrists, Neurobiologists, and the National Security community. To quantify and analyze these dynamics, Mathematicians typically apply tools from probability and statistics, logic, and even differential equations, culminating in big data analytics. However, the "psychological-space" is not, in general, metricizable. This topological treatment extends mathematical-psychological analysis to the study of properties of psychological structures and maps. Covariance between cognitive states and perceived changes in the physical environment suggest mappings between psychological and physical spaces. How do we define continuity properties for these maps without metrics? In this theoretical research, we state the structures and dynamics of psychological phenomena in purely topological terms. In particular, a topological base is defined for cognitive substructures and for physical world environments, without the constructs of quasi-metrics, pseudo-metrics, or metrics. We topologically describe continuity properties for perception and behavior maps between the physical world and cognitive substructures. Cognitive covering-spaces are also defined for physical world environments. Also, motivated by the minds ability to analyze and separate sets of experiences, this work explores implications of separation axioms on cognitive substructures, with respect to semantic associations of mental data. Finally, we explore a Post-Traumatic Stress symptom which suggests an inverse relation between emotional intensity and the "Hausdorff-ness" of cognitive substructures. We provide many examples to support and explicate this theoretical work.

Keywords*:* *Mathematical-Psychology, Topological-Psychology, Cognition, Continuity, Covering Space, Non-Hausdorff Space, Stone Topology, Separation Properties*

INTRODUCTION

Mental structures and dynamics do not have the typically well-defined and well behaved metrics, as metrics for the physical world. In this research, we state the structures and dynamics of psychological phenomena in purely topological terms. Our approach admits a qualitative mathematical analysis on mental substructures. Kurt Lewin (1936), a twentieth century social psychologist, stated in his book Principles of Topological Psychology that the physical sciences have taken the opportunity to apply mathematics to describe physical properties and laws

(Lewin, 1936). According to Lewin, Topology is also a suitable theory for the scientific study of psychological phenomena. In (Lewin, 1936) Lewin defines a behavioral equation, $= f(P, E)$, expressing a resultant behavior as the person P and environment E interact, where the person and the environment are part of a psychological-topological “Life-Space”. He defined regions, boundaries, points, and movement in a dynamic psychological space. Lewin introduced the concept of psychological force field analysis, connecting topology and vector psychology (Włodzisław, 2017).

Inspired by Lewin, Ivancevic and Aidman (2007, p. 616) apply generalized Feynman path integrals on this dynamic psychological space, which they call a “Life-space Foam” (LSF). Their life-space foam is macroscopically represented as a smooth manifold of steady force-fields and behavioral paths, and microscopically as a collection of fluctuating force-fields, locomotion paths, where local topologies can have holes. While metrics on the LSF for the generalized Feynman path integrals are not explicit in (Ivancevic & Aidman, 2007, p. 616), they were able to capture the psychological dynamics involving motivation, intention, and decision under the principle of least action, where the action integral is defined to be effort.

Falmagne and Doignon (2011, p. 23) define cognitive entities as Knowledge Structures and Learning Spaces; combinatorial structures describing the possible states of knowledge of a human learner. These Knowledge Structures are subset systems of knowledge content, where a subset of knowledge elements is called a Knowledge state. Falmagne and Doignon apply knowledge-space theory to explore the dynamics of learners navigating through selected topologies on knowledge content. Danilov (2009, p. 510) discusses properties and gluing theorems of Knowledge spaces in terms of pre-topologies, antimatroids, topologies, and filters. Motivated by the minds ability to analyze and separate sets of experiences, our research explores implications of separation axioms on cognitive substructures, with respect to semantic associations of mental data.

Mental “Objects”

The question is, “what are the points, or primitives, of the psychological-space?” Eysenck and Keane (2000, p.266) view the psychological space as a space of “mental representations”. That is, the “points” are mental representations of aspects of “things” from the physical world or aspects of “things” imagined. They consider two types of mental representations: analogue representations of things such as physical objects, and symbolic representations of say written language. Symbolic mental representations are propositional and capture ideational content, while analogue mental representations hold sensory data (visual, auditory, kinetic, odor, taste). Also, there are some differences in “mental-grammar” on how the two types of mental representations are combined to make sense. For propositional representations of a language there are strict combination rules. There is usually a strict grammatical method for combining symbols (letters, words, characters, operations) so that a combination makes sense. Many sentences that make sense, will not make sense if the words are permuted or the words are not punctuated by spaces.

The "mental-grammar" rules are not as strict for combining the analogue representations of entities perceived from the physical environment (Eysenck & Keane, 2000, p.266). For example, the "mental-grammar" for combining mental representations of the taste of an apple, the image of an elephant, and the sound of that elephant. All modes of mental representation would be properly combined, under some "mental-grammar", for a person eating an apple at the zoo while seeing and hearing an elephant. We consider a cognitive subspace of the mind to be a space of mental representations of aspects of "things" from the physical world or aspects of "things" imagined, as in (Eysenck & Keane, 2000, p.266).

According to Adler and Rips (2008, p. 878) cognitive subsidiary systems manipulate the representations solely by virtue of the form (mode) of the representations in which they are couched. Environmental information is expressed in the mind as symbols, in whatever format (mode and syntax) that the cognitive process demands of the representations that they apply to. Comprehension of environmental information not only involves "mental transduction" into mental symbols, but also semantics and some mental-syntax (Adler & Rips 2008, p. 878). At best, we describe this cognitive space as a generalized algebraic mental substructure $\mathcal{C} = (C, \sigma, I)$ having a set of symbols (mental representations) C, a signature σ that describes some generalized "mental-grammar" concatenation rules on C, and an interpretation semantic function I that gives meaning to representations and concatenation. For the mind, the interpretation function I, combined with the concatenation rules, aids in rational negotiation between the mind and the physical world, or negotiation between the mind and itself or other minds.

Psycho-Physical Covariance

A subsidiary cognitive mental system is described in (Adler & Rips 2008, p. 878) that is responsive to the flow of environmental events in the physical world. Adler and Rips consider the mind to be a symbol manipulating device where much of the psychological structure is constituted of mechanisms that mediate exchanges between the mind and the physical environment. The states of these subsidiary cognitive systems co-vary with environmental states. Covariance is spatial/temporal. The subsidiary cognitive systems affect overall cognition in ways that are responsive to the spatial/temporal flow of environmental events (Adler & Rips 2008, p. 878). Covariance between cognitive states and perceived changes in the physical environment suggest mappings between psychological and physical spaces. We topologically describe the continuity properties of perception and behavior maps between the physical world and cognitive substructures.

Emotion and Cognition

When looking at cognitions and emotions, we tend to view them as separate entities, possibly interacting but for the most part disjoint. We as individuals would not be able to navigate our respective realities without both cognition and emotions. The mental facets of emotions play a role in our decision making and act as a beneficial guide to the individual as they traverse their own respective reality. However, we also know that the individual would not be able to properly navigate their own physical environment, R3(t)-space, without the proper pairing of both emotions and cognitions. Here, R3(t) is the dynamic 4-dimensional physical time-space world.

The concept of emotion pairing with cognitions has been initiated by Izad (1992, p. 561) with the ideology of combining feeling and thought, through "affective-cognitive" structures. Whenever an emotion becomes linked to any mental image, symbol, or cognitive thought associated with an experience from the environment, immediately a feeling develops, or more definitive, an affective-cognitive structure develops (Izad, 1992, p. 561). Emotions play specific roles in our behavior in the physical world, whether stimulated from with-in or without the person. Many theoretical ideologies and schools ground themselves in the idea that emotions are specified modes of operations which are solely shaped and created by natural selection; stimuli from the physical environment (Plutchik, 2003). Even though this theory is widely accepted, the set of modes of operation (emotions) are not given a "structure" in the mind.

From a psychological basis, emotions are a complex state of feeling that results in physical and psychological changes that influence thought and/or behavior, while some mathematical scholars look at emotions as a pervasive component of human experiences (Eligio, 2017). Sims (2011, p. 29) described any "philosophy" in a person's cognitive space as an abstract simplex spanned by its principles, where each principle-coefficient in the philosophy is a sentiment-value, in the interval $[0,1]$, subject to the person's preferential prioritization of the spanning principles. With that basis established, emotions are an integral part of the physical-mental-space relation. The six primary emotions established by psychologists Paul Ekman and Wallace V. Friesen are Anger, Fear, Surprise, Happiness, Sadness, and, Disgust (Ekman & Anoli, 1980, p. 1125). These six fundamental emotions, placed together in any combination desired, gives a new emotion, e.g., joy, trust, anticipation, calmness, courage, rage, grief etc.

There are many speculations, one being that emotions and cognition occur in the same mental space. Does the process of cognition occur with emotion simultaneously? Cognitive processes can occur without the addition of emotion; while, emotions can influence every substructure of the mind-space. We can only conclude that the class of emotions acts as a distinct substructure of the mind. We further conclude that one, if not more, of these emotions "arise" during a cognition process, influencing both the thought process and the behavior in the physical environment.

Emotion and "Hausdorff-ness" in the Mind-Space

Hausdorff spaces are topological spaces in which any two distinct points can have disjoint neighborhoods, respectively. Theoretically, this space has a topology producing a separation property capable of distinguishing distinct points. More specifically, whenever a and b are distinct points of a particular set, X, there will exist disjoint open sets Ua and Ub, such that Ua contains a and Ub contains b.

We consider the elements (points) of the cognitive space to be mental representations of some aspects of R3(t)-space, and/or aspects of the mind itself (Eysenck & Keane, 2000, p.266). Also, we can safely say that, in general, some thought collections in the mind are not likely to be perceived in terms of their distinct "internal-points"- that are contained in disjoint sub-collections (neighborhoods). Those types of thoughts, "internal-points", are almost inseparable in the Mind-Space, unless acted on by the person. Now, in-regard to the emotions, we consider that emotions can pair with cognitions in a mental cross-product structure A X C, where A and C are emotional and cognitive mental substructures, respectively. In this product, the effect of emotion and cognition do not readily separate. In general, it is difficult to separate the emotion from the cognition. This pairing creates a feeling, possibly conveyed through physical, biological, chemical and/or even neurological behaviors.

Emotions can affect the topological separation properties of mental cognitive substructures. Consider a veteran soldier who experienced a traumatic event,
$\boldsymbol{E} \subset \boldsymbol{R^3(t)}$, during a war, in the past, and executed a survival tactic to survive. The emotional trauma can "solidify" an image" (mental representation), $\mathcal{E} \subset \boldsymbol{C}$, of the traumatic event as a subset of that veteran's cognitive substructure $\boldsymbol{C}$. Any "harmless" post war event resembling an element in E can stimulate a survival behavior $B = f(\mathcal{E})$ in R3(t)-space, as a reflex to the entire mental subset $\mathcal{E}$. In other words, the elements of $\mathcal{E}$ are indistinguishable to the veteran, with respect to behavior. Thus, $\boldsymbol{C}$ is a non-Hausdorff cognitive substructure since it contains the set $\mathcal{E}$, whose elements are not separable, by the veteran.

We can infer that as an individual experiences an extreme of one of the six primary emotions, or any combination of them, due to an event, that extreme emotion can "bind" the mental-images of that event into "indistinguishable" points, with respect to behavior. We further conjecture that the mind-space becomes "less Hausdorff" with increase in emotional intensity, above some traumatic threshold. Emotions and "Hausdorff-ness" are revisited in section 5.

1. COGNITIVE SUB-STRUCTURE OF THE MENTAL SPACE

To create a Mathematical structure for Mental Cognitive processes we first need to have a mathematical understanding of a concept which we call a Sub-Mental structure (C) of the Mind-Space. Spaces are important to mathematics because many of our math theories are based on spaces that have innate properties. Spaces provide the foundation for us to formulate mathematical structures, what you can do in them, and what is possible. The reason for the use of a mental sub-structure is because of the nature of the Mind as a space. There is no perceivable boundary to the mind. A boundary of a set consists of points such that, if you were to put any neighborhood around any boundary point, that neighborhood would contain some points inside and outside of the set. Take this definition with respect to the Mind-Space; there are no mental points (representations) or physical "things" outside of the set (Mind-Space) to consider as boundary points, because the Mind-Space is not taken to be submerged in any encompassing set. We can find no neighborhoods of mental points (representations) or physical "things that would intersect both the mental space and physical world. Further, in the Mind-Space, we assume that we are working with a no time-space condition, no distance, no shape, and no angles, since there is no conventional distance between thoughts; and also, no geometric shape for a thought or a collection of thoughts. Since the Mind Space does not have the typical properties found in the physical world, we employ a Topological treatment to the study of cognitive psychology to provide a qualitative mathematical analysis of the mind.

Using generalized concepts in Topology, we can identify properties of Mental substructures mathematically. Call this mental substructure of the mind the cognitive subspace C. The basic element (point) of the Mental space is a thought. That thought is a mental representation (image) of something from some perceptual physical environment in R3(t) or a mental representation of something brought about by the imagination process of the mind itself [6]. The ability to combine thoughts to produce other thoughts or use thoughts to infer other thoughts suggest that the mind has more structure, some type of generalized signature. The mental cognitive substructure $\mathcal{C} = (C, \sigma, I)$ has a set of symbols (mental representations) C, a signature σ that describes some generalized "mental-grammar" concatenation rules on C, and an interpretation semantic function I that gives meaning to representations and concatenation.

We note that, in the mind, the thoughts (mental representations) can be considered as elements or sets, so that "contained in C " can be shown by ∈ or ⊂. The notion of primitives is taken up in a discussion on "pointless topology" in (Johnstone, 1983). In this most general mental setting, the mind may use, as a primitive, what we call a "point", a "subset", a "class of subsets", a "filter", or even whole topologies as a primitive.

Cognitive Closure property

If the union and intersection operations are elements of the signature , there are some mental closure properties. Take the union of any two arbitrary thoughts in C, $a_n, a_j \in C$, you would come up with another thought construct $\{a_n\} \cup \{a_j\} \subset C$, which is contained in C; and, if we take the intersection (commonality between thoughts) of any two arbitrary thoughts, $\{a_n\} \cap \{a_j\} \subset C$, that will also be another thought contained in C. Even if there is nothing in common between the thoughts under observation, the concept of "nothing in common" is also a thought in C. Thus, C is closed under finite union and intersection of thoughts, concepts, constructs, or ideas. In general, for any mental "combination rule" $* \in \sigma$, then $a_n * a_j \in C$. The mental substructures, C, are generalized algebras resembling mental mathematical magmas.

Cognitive Sequences

Formal Sequences also take place in the mind, such as a "train of thought". A person can take a thought $a_0 \in C$, and from it derive a finite sequence of thoughts $\{a_i\}_{i=0}^{n} \subset C$ that leads to finding another thought $a_n \in C$, the limit of the sequence. Note that the sequence (train of thought) may or may-not be a directed set in the cognitive space C. Take for example a person that has an idea for an invention they want to create. They have an initial idea, call this idea a_0; through critical thinking ($\{a_i\}_{i=0}^{n} \subset C$) and trial and error of using ideas a_i, the person can finally come to a "satisfied" idea, called it a_n, a clearer understanding than their initial understanding of the perceived idea. This sequence shows a complete process in which there is a limit. Each idea a_i is related to the other in consideration to what the person is using them for, and the sequence contains its limit. Taking this into account we can call this sequence of ideas a Closed set with respect to limit points; and, in Topology if a set is closed, that means its complement is open. The complement in this case would be the rest of the cognitive space, $C - \{a_i\}_{i=0}^{n}$, which is Open.

On the contrary, take a “train of thought” $\{a_i\}_{i=0}$ that has no conclusive end. Meaning that, in that person’s mind, the limit for the sequence is never reached. Either there exists a limit (conclusion) that the person cannot comprehend or the sequence of thought may lead to more than one limit (the limit is not unique), where the person becomes indecisive. In one sense, the sequence $\{a_i\}_{i=0}$ not containing its limit point cannot be said to be closed, but open. Therefore $C - \{a_i\}_{i=0}$ is closed. In another sense, if all the a_i in the sequence are isolated points with respect to some topology, then the sequence is closed, therefore $C - \{a_i\}_{i=0}$ is open. Since $C - \{a_i\}_{i=0}$ can be open and closed we will use the term clopen. Clopen is a property that is shared by other known sets such as the real number system (R), and the empty set Ø. Having this initial mathematical understanding of a mental substructure, C, we will now move on to the relationship between the cognitive space C and the perceptual physical world $R^3(t)$. Recall that R3(t) is the dynamic 4-dimensional physical time-space world. While the time-dependent spatial coordinates $(x(t), y(t), z(t))$ can be the typical “points” in $R^3(t)$, our topologies on $R^3(t)$ will not involve metrics on subsets of such points; so, without loss of generality we will not employ the use of coordinates $(x(t), y(t), z(t))$.

2. TOPOLOGICAL BASES UNDER PSYCHOLOGICAL CONSIDERATION

2.1 A Topology on Perceptual Environments in $\boldsymbol{R^3(t)}$

Topologies on environments in $R^3(t)$ are a cognitive consideration by individuals. Entities (objects or actions) in the perceptual physical world, $R^3(t)$, are consider relevant, valuable, or practical to an individual or individuals if those entities satisfy a purpose in the minds of those individuals. Sets of objects or events can be used to create a topological base for the perceptual physical world, by considering whether or not an object/event or set of objects/events satisfies as complete and practical for use, in the mind of a person or group of persons. Environments are topologized by people. The assignment of topologies on a set is subjective.

Definition 2.1.1

(Neighborhood) In the psychological sense, we define a neighborhood $K(x)$ of an element x in $R^3(t)$ to be “a set of parts as a practical working whole” that contains x. Here, the set K of parts in $R^3(t)$ is considered to be a neighborhood of any of its “parts” if and only if that set K as a whole, satisfies a practical need of a person.

In the following definitions, we take all environments E in $R^3(t)$ to be finite classes of objects-matter or actions.

Definition 2.1.2:

(Topological Base) In some environment E of $R^3(t)$ let B be a collection of objects $x \in E$, where x is either complete but not a practical whole, or an irreducible set that is a practical whole (pw). B is a base for a neighborhood system in E if and only if every practical whole K in E contains some object(s) x in B.

Definition 2.1.3:

(Complete Object) A complete object in $R^3(t)$ is an object that is considered to be whole, a unit, with respect to some psychology.

Definition 2.1.4:

(Irreducible Set) An irreducible set is a connected set that is not the union of any other complete sets.

Definition 2.1.5:

(Open Set) Let B be defined as in Definition 2.1.2. A set K is open if and only if K is a finite union of objects from B and K is a practical whole (pw), e.g., $K = \bigcup B_i$, for $B_i \in$ L, where L is some finite subset of B.

Note: (Spatial/temporal Proximity) For most practical purposes, in order for the "neighborhood" or "union", defined in definitions 2.1.1 and 2.1.5, to have any real meaning to the individual involved, some spatial/temporal proximity requirements for objects in the "neighborhood" or "union" must be met. Henceforth, we consider all temporal/spatial proximity requirements as satisfied when referring to "neighborhoods" or "unions" in $\mathrm{R}^3(\mathrm{t})$. For example, if we consider a pair of shoes as the disjoint union of a left and a right shoe, we will always require that both shoes are near each other, in time and space, for practical use.

In the most general sense, in $\mathrm{R}^3(\mathrm{t})$, an element in the base B can be a complete object having mass, or a complete action, or a behavior necessary to fulfill a task. Thus $x \in$ B can be a physical object or an action. In this sense K, as defined in definitions 2.1.1 and 2.1.5, can be an event composed of physical objects and actions, where K fulfills a practical need of a person or a group of people. The practical whole K can even be a musical score.

Example 2.1.6.

In $\mathrm{R}^3(\mathrm{t})$, a wedding event, W, can be considered as a practical working whole by a bride and groom, where W is the union of base elements: bride, groom, obtaining of marriage license (action), priest/imam/rabbi, location, taking of vows (action), signing of marriage license (action), obtaining cake and food (action), cake, food, guests, and music. By definitions 2.1.1 - 2.1.5, W is an open neighborhood consisting of base elements, where each element is complete and is either an object (person, place, or thing) or an action. Note that each person is an irreducible base element.

Example 2.1.7.

In $\mathrm{R}^3(\mathrm{t})$, a pair of shoes N is a practical whole that is a disjoint union, where the pair is a neighborhood of any of its shoes, whether the left shoe "l", or the right shoe "r", written

N(l)=N(r)= {l, r}.

The sets {l} and {r} are complete elements in some base B. The pair of shoes N is a practical working whole; but, N is a disjoint union of complete base elements, and is not an element in any base B by definition 2.1.2.

Example 2.1.8.

A box $\mathrm{q} \in \mathrm{R}^3(\mathrm{t})$ is a practical whole that is a connected set, and is its own neighborhood N(q)={q}. Since the box is an irreducible practical whole, it is a base element in some base B.

Note that for all practical containment use, the box does not remain practical as a physically disjoint union of its subsets (sides). The pair of shoes N(l) must remain a disjoint union of its subsets {l} and {r} to remain a practical whole.

Example 2.1.9

. A dining-room set D of six chairs and a table serves as a disjoint practical whole $\mathrm{D} \in \mathrm{R}^3(\mathrm{t})$. We can write $\mathrm{D} = \{\mathrm{t}, c_1, c_2, \dots, c_6\}$ where D is also a neighborhood of any of its parts

$$\mathrm{D} = \mathrm{N}(\mathrm{t}) = \mathrm{N}(c_1) = N(c_2) = \cdots = N(c_6).$$

Each part in D, on its own, is also an irreducible practical working whole- the table, and each of the chairs separately- and are elements of some base B. By definition 2.1.1, the table is a neighborhood of itself N(t)={t}, and so is each chair a neighborhood of itself, N(cn)={cn}, since they each can serve alone for practical use. The table is a connected set and each chair is a connected set, and must remain connected for practical use, so by definition 2.1.2, they are elements in some base B. While the dining-room set D is a practical working whole, it is also a disjoint union and is not a base element by definition 2.1.2.

Example 2.1.10.

A car tire is not considered as a practical working whole, since it is not commonly used alone to serve a purpose – "we do not drive a tire to travel to the store". The tire must work with other parts in $\mathrm{R}^3(\mathrm{t})$. By definition 2.1.1, the tire is not a neighborhood. It is only a base element. The car is a neighborhood of the tire or any other parts of itself.

Example 2.1.11.

A combination code C of numbers, to logon to a website, is an ordered practical working whole. C is a neighborhood of its elements, with each element being in some base B. If the order of elements is changed, the new ordered combination C' has the same base elements as C, but will not open the website, so C' is not a neighborhood and not considered an open set by definition 2.1.5.

Example 2.1.12.

Some cooking recipes, R, are ordered practical working wholes. Similar to example 2.1.11, if R' is a reordering of recipe R, and produces an undesirable outcome, then R' is not a neighborhood and not considered an open set.

Definition 2.1.13. (The Practical Topology)

Let E be some environment in $R^3(t)$.
Let B $= \{B_i |\ B_i\ is\ complete\ or\ a\ connected\ pw\ in\ E\}$.
Let $K_j = \bigcup B_i$, for $B_i \in$ L, where L is some finite subset of B, and K_j is a practical whole in E. Consider the collection of sets $T = \{\emptyset, E\} \cup \{K_j\}$, then T is a topology on E and B is a base for T.

Definition 2.1.14. (Closure)

Motivated by examples 2.1.6 – 2.1.12, we define the closure of any base object, $\{B_n\}$, to be the practical-whole union, of which B_n is a part of; that is, $\overline{\{B_n\}} = K_j = \bigcup B_i$, where K_j is a practical whole. In this sense, each B_i is a "limit object" of K_j. Here, limit objects are defined to be those objects necessary in a union to make that union a practical whole (pw).

Proposition 2.1.15.

By definitions 2.1.5 and 2.1.14, the practical wholes, $K_j = \bigcup B_i$, in the topology defined in definition 2.1.13 are both open and closed (clopen).

Definition 2.1.16.

(B -Semi-open)Let X be a topological space. A subset $A \subset X$ is said to be B -semi-open, with respect to base B if there exists a set $V \in$ B such that $V \subseteq A \subseteq \bar{V}$. This definition is a specialization of the definition for semi-open set in (Neeli, 2012, p. 121; Missier & Jesti, 2015, p. 27).

Proposition 2.1.17.

The practical wholes, $K_j = \bigcup B_i$, in the topology defined in definition 2.1.13 are B -semi-open.

Definition 2.1.18.

(Practical Equivalence Classes)For every object a in $\mathrm{R}^3(\mathrm{t})$ that is part of a practical whole, we consider the practical whole to be a neighborhood of a, $\mathrm{N}(a)$. $\mathrm{N}(a)$ is also an equivalence class of a; $\mathrm{N}(a) = \{\, r |\ r \sim a\}$, where $r \sim a$ if and only if r and a contribute to the same working practical whole.

Proposition 2.1.19.

An environment $E \subset \mathrm{R}^3(\mathrm{t})$ having the "Practical Topology" defined in definition 2.1.13 is an R0 topological space. R0 topological spaces are revisited in section 5.

Proof: For any $x, y \in E$. Let y be an object in a practical whole K and x be in the closure of y, $x \in \overline{\{y\}}$. Since the practical whole K is also an equivalence class and neighborhood of y, $K = \mathrm{N}(y)$, then x and y are in the "practical" equivalence relation $y \sim x$, so that y is also in the neighborhood of x , $y \in \mathrm{N}(x) = K$, thus $y \in \overline{\{x\}}$. Hence E is an R0 topological space.

2.2 Cognitive Points

Falmagne and Doignon (2011, p. 23) explore cognitive entities as Knowledge Structures and Learning Spaces; combinatorial structures describing the possible states of knowledge of a human learner. They investigate the knowledge content and the cognitive arrangement (subsets) of such knowledge content in a student's mind, pertinent to problem solving. Danilov (2009, p. 510) presents these cognitive arrangements on knowledge spaces as Pre-topologies and Antimatroids with certain gluing criteria for gluing such spaces. A Pre-topology is a topology where the finite intersection property is relaxed. An Antimatroid is a pre-topology T with the property that, for every nonempty open set $V \in$ T, there exist $x \in V$ such that $V - \{x\}$ is also open.

Certainly, what is important to any individual is the type of cognitive data (knowledge) in the mind, how that individual interprets that knowledge, and how they use that knowledge, via some form of concatenation and implication to negotiate in life. We consider the cognitive subspaces of the mind to be spaces of mental representations of aspects of "things" from the physical world or aspects of "things" imagined, as in (Eysenck & Keane, 2000, p.266).

As a generalized algebra, the cognitive substructure $\mathcal{C} = (C, \sigma, I)$ has for its objects the mental representations of statements (formulas) that can be many-sorted formulas, where the symbols of a formula can be of different sorts. Consider a mental formula written as

$$\phi_1 : x_1 \ likes \ x_2.$$

In the formula ϕ_1 the domain for x_1 will typically be a mental representation of a person or animal in R3(t), while x_2 can be a mental representation of anything that can be perceived from a domain in R3(t). In formula ϕ_1 both x_1 and x_2 are mental representations. Also, "likes", here, is a mental representation- a relational predicate, a concept, as in (Eysenck & Keane, 2000, p.266). x_1 and x_2 and "likes" are of different sorts.

If x_1 and x_2 and "likes" are taken to be primitives (elements) in the formula so that $x_1 , x_2 , likes \in C$, then we can consider the mental formula ϕ_1 to be a subset of C, with a syntax σ and interpretation I on it; thus $\phi_1 \subset C$. If we consider a Filter on C where ϕ_1 is taken as an element of a member B of the Filter, $\phi_1 \in B$, then we have the relation $\phi_1 \in B \subset C$. So that ϕ_1 is a primitive in C.

Clearly, there are many formulas (phrases, statements, commands, or interrogatives) that are mental representations in and formed by the cognitive mind; for example

$\phi_2: x_1\ was\ studying\ x_2.$, $\phi_3: x_1\ and\ x_2\ did\ a\ corporate\ merger.$,
$\phi_4: Dogs\ are\ animals.$ $\phi_5: Mice\ eat\ cars.$, $\phi_6: Purple.$, $\phi_7: Hard.$, $\phi_8: Computer.$,
$\phi_9: Walk.$, $\phi_{10}: Walking.$, $\neg\phi_{10}: Not\ Walking$ $\phi_{11}: Walked.$,
$\phi_{12}: I\ understand\ the\ story.$, $\phi_{13}: I\ feel\ happy.$, $\phi_{14}: Tell\ x\ to\ rake\ the\ yard.$, or
$\phi_{15}: How\ much\ does\ x\ cost?$, etc.

Also compound statements and implications:
$\phi_{15}: x_1\ and\ x_2\ did\ the\ corporate\ merger, while\ x_3\ secured\ the\ funds.$, or
$\phi_{16}: if\ there\ is\ enough\ funding, we\ can\ move\ ahead\ with\ the\ construction.$

A mental formula ϕ can be a whole proof, argument, story, treatise, or philosophy in an individual's mind. A basic formula can also be a mental image (representation) of an airplane, odor of a rose, texture of cotton, taste of a peach, or sound of a violin.

2.3 Cognitive Subset Systems and Topologies

For practical purposes, it may be important for a person to parse their mental formulas into subsets of information in-order to solve a problem or negotiate a situation. Certain classes of subsets, called filters, can help organize ones thinking into sequences of thought that leads to solutions. A person may organize subsets of formulas into topologies, pre-topologies, or just classes of subsets to accomplish a mental or physical task. During "trial and error", a person may organize a collection of subsets of formulas that they believe will help them accomplish a particular task.

Definition 2.3.1.

(Filter) Let X be a nonempty space. A filter F on X is a collection of subsets of X such that
(1) for any sets $A \in F$ and $B \in X$, if $A \subset B$ then $B \in F$, and
(2) if any sets $A, B \in F$, then $A \cap B \in F$.

Definition 2.3.2.

(Psychologically Significant Set) Given a cognitive substructure
$\mathcal{C} = (C, \sigma, I)$, where $C = \{\phi_n\}$ is a set of formulas (mental representations), a subset $A \subset C$ is called psychologically significant with respect to, $\phi_* \in A$, if A is a set of formulas $\{\phi_{*_i}\}$ that combine under some "mental grammar" so that ϕ_* makes sense in the mind or in the real world-so that ϕ_* is relevant in the mind or in the real world.

The set of formulas $\{\phi_{*_i}\}$ makes another formula $\Psi_* \in A$, under generalized n-nary mental functions f^α in σ, where ϕ_* is the subject of the formula Ψ_*.

$f^{\alpha}: \prod_i^{\alpha} \phi_{*_i} \rightarrow A$, where α is the arity and Π is the cross product of formulas. The generalized functions f^{α} are such that they act on mental representations defined by some "mental grammar" under the interpretation function I.

Consider the formula of a mental representation of a lion, ϕ_*. There can be sets containing ϕ_* that are psychologically significant. For instance, $\phi_* \in A$, where A is the set of mental formulas organized to study, ask questions, and do research on lions. We can also have $\phi_* \in B$, where B is the set of mental formulas organized to protect oneself from lion.

Definition 2.3.3

. Let C be a cognitive subspace of mental representations of statements $\{\phi_n\}_{n=1}^{k}$. Define the set $\widetilde{\phi_n} = \{A \subset C \mid \phi_n \in A\}$, the set of all subsets of C such that ϕ_n is an element of A, and A is "psychologically significant" with respect to ϕ_n. $\widetilde{\phi_n}$ is a filter on cognitive space C.

We can also have filters associated with more than one statement, for example
$\widetilde{\phi_{nm}} = \{A \subset C \mid \phi_n, \phi_m \in A\}$, or $\widetilde{\phi_{nmp}} = \{A \subset C \mid \phi_n, \phi_m, \phi_p \in A\}$, etc.

$\widetilde{\phi_n}$ is an ultrafilter on cognitive space C; while, $\widetilde{\phi_{nm}}$ is a principle filter of ϕ_n and ϕ_m but is not an ultrafilter, since it can be extended to the filter $\widetilde{\phi_n}$. What is interesting in the cognitive sense is that a subset A "associated" with statement ϕ_n could be any selected sequence of statements in a "train of thought" that concludes with or begins with ϕ_n; that is, A could be some directed set $\mathbf{A} = \{\psi_j\}_{j=1}^{q}$, where
$\psi_1 \rightarrow \psi_2 \rightarrow \cdots \rightarrow \psi_q = \phi_n$, or $\phi_n = \psi_1 \rightarrow \psi_2 \rightarrow \cdots \rightarrow \psi_q$.

Note: We also admit the possibility that a statement ϕ_j and its negation $\neg\phi_j$ can both occur in the same cognitive subset $A \subset C$; since, contradiction in the human mind is possible.

Definition 2.3.4

. (A Cognitive topology). Let $\boldsymbol{B} = \{\widetilde{\phi_n}, or\ \widetilde{\phi_{nm\ldots p}}\}$. Consider the collection of subsets of C; M $= \{ M_j \subset C \mid M_j = \cup \tilde{B}\ for\ some\ \tilde{B} \in \boldsymbol{B}\}$. M is a topology on C with base $\boldsymbol{B}$. Here, the M_j are considered open in C, and so are the $\tilde{B}$'s.

Definition 2.3.5.

(The Stone Topology) In a cognitive dynamic, consider a level of classification of information of formulas where the ultrafilters are the "points"; that is, the primitives of a cognitive substructure $S(C)$ are ultrafilters $\widetilde{\phi_n}$.
Define $\hat{Y} = \{\widetilde{\phi_n} \in S(C) \mid Y \in \widetilde{\phi_n}\}$, where Y is a subset of formulas in C. Here, the ultrafilters $\widetilde{\phi_n}$ are "large" associations of mental data that are taken as units of information in the mind. A base for $S(C)$ given by the collection B $= \{\hat{Y}\}$ is called the Stone base, and $S(C)$ is the Stone space associated with C (βS as a topological space, ret. 2017).

Proposition 2.3.6.

If the base is a collection B $= \{\hat{Y}\}$ then the topology on $S(C)$ generated by B is the Stone Topology (βS as a topological space, ret. 2017). It is well known that the space $S(C)$, with respect to the Stone Topology, is Hausdorff, Compact, and totally disconnected. That is, all subsets of $S(C)$ are clopen under the Stone Topology.

Mental Predicate Space

Associated with perceived physical environments, events, and objects in $\mathrm{R}^3(\mathrm{t})$ are predicates in the mind, that the mind uses to describe some aspects of the perceived environment (Eysenck & Keane, 2000, p.266). Formally a predicate is the part of a statement having a verb, and describes something about the subject. Here, we take a predicate to be any mental representation that the mind uses in its formulas to identify, describe, and make sense out of an entity from $\mathrm{R}^3(\mathrm{t})$ or the mind itself. We also allow mental representations of single adjectives to be predicates. Topological predicates are discussed in (Pauly & Schneider, 2006).The predicates can describe properties, qualities, conditions, and relational aspects between objects. In general, the predicates can also describe conditions of the mind such as feelings, levels of understanding, etc. As in (Eysenck & Keane, 2000, p.266), we consider the class of mental representations of predicates in the mind. The class of mental representations of predicates P is a cognitive subspace of the mind M.

Definition 2.3.7

. (Predicate Base) A base for the mental predicate subspace P of M is the class of all distinct sets of mental representations of predicates describing a distinct aspect of a "thing";

$$\mathbf{B_P} = \{\mathrm{P_i} | \mathrm{P_i} \text{ is a disntict set of predicates } \},$$

where each $\mathrm{P_i}$ is a neighborhood of some predicate $\mathrm{p} \in \mathrm{P_i}$ that describes a feature or aspect of some object or event in a perceived environment in $\mathrm{R}^3(\mathrm{t})$, or a perceived condition of the mind.

We note that there is the possibility that a person may not have predicates in their memory or cognitive space to identify with an entity, say $k \in \mathrm{R}^3(\mathrm{t})$. In that case k could map back to the ubiquitous empty set $\emptyset \subset \mathbf{P}$, or the person may try to associate k with something that they are familiar with.

Example 2.3.8.

Some distinct sets of mental representations of adjectives, that we will call predicates in the mind:

$\mathrm{P}_1 = \{the\ set\ of\ colors\}$; $\mathrm{P}_2 = \{the\ set\ of\ shapes\}$;
$\mathrm{P}_3 = \{the\ set\ of\ prepositions\}$;
$\mathrm{P}_4 = \{the\ set\ of\ texture\ characteristics\}$;
$\mathrm{P}_5 = \{the\ set\ of\ sound\ characteristics\}$;
$\mathrm{P}_6 = \{the\ set\ of\ visual\ characteristics\}$;
$\mathrm{P}_7 = \{the\ set\ of\ taste\ characteristics\}$;
$\mathrm{P}_8 = \{the\ set\ of\ odor\ characteristics\}$;
$\mathrm{P}_9 = \{the\ set\ of\ emotional\ characteristics\}$;
$\mathrm{P}_{10} = \{the\ set\ of\ intellectual\ characteristics\}$.

Since a person may describe an environment or their own state of being in any creative way, the list of predicate sets in example 2.3.8 is not exhaustive. If we consider only distinct sets of predicates, then any pair of sets is disjoint, $\mathrm{P}_\mathrm{j} \cap \mathrm{P}_\mathrm{m} = \emptyset$ for all $j \neq m$, where each P_i is a neighborhood of some predicate p or contains a neighborhood of p.

An element p in any predicate class P_i , used by two persons to describe the same object, may differ in degree with respect to the perception of each person. For example, it is possible that person A and person B observing a red apple, can perceive the "redness" different from each other, while they both agree that the apple is red; that is, they do not see red the same way. This suggest that there is a range of "redness" that the two persons are perceiving in.

Definition 2.3.9

. (Open predicate neighborhoods) We will consider any perceptual range or degrees of a mental predicate representation p to be an open neighborhood $V_\mathbf{p}$ for that predicate p.

Example 2.3.10

. Refer to Example 2.3.8:

(i) If p $\in \mathrm{P}_1$ is some mental predicate representation for red, p = red, then there is some perceptual range of "redness" that p is a member of, call it the open neighborhood V_r.

(ii) The mental predicates for saltiness s $\in \mathrm{P}_7$, roughness h $\in \mathrm{P}_4$, or timbre t $\in \mathrm{P}_5$, will be in the perceptual ranges (open neighborhoods) $V_\mathbf{s}$, $V_\mathbf{h}$, $V_\mathbf{t}$, respectively.

(iii) If c $\in \mathrm{P}_2$ is some mental predicate representation for circular shape, c = circular, then there is some perceptual range of "circularity" that c is a member of, call it $V_\mathbf{c}$. Here, the degree of circularity of a vitamin pill or car tire, perceived by any one person, will place the shape predicate in the open neighborhood $V_\mathbf{c}$.

(iv) If $j \in P_9$ is some mental predicate representation for a joyous feeling, j = joy, then there is some perceptual range of "joy" that j is a member of; this will be the open neighborhood $V_{\mathbf{j}}$. The degree of joy of a person can vary in $V_{\mathbf{j}}$.

3. THE MIND-$\boldsymbol{R^3(t)}$ RELATION AND CONTINUITY

The ability for an individual to be able to think in a mental space $\boldsymbol{M}$ (their mind) and execute that thought as behavior in the physical world $\mathrm{R}^3(\mathrm{t})$, motivates the consideration of a map $\boldsymbol{f}$ from a cognitive subspace $\mathrm{C} \subset \boldsymbol{M}$ into $\mathrm{R}^3(\mathrm{t})$. The map $\boldsymbol{f}$ models a mental process expressed as behavior in the physical world $\mathrm{R}^3(\mathrm{t})$. A map $\boldsymbol{g}$ from $\mathrm{R}^3(\mathrm{t})$ to some cognitive substructure $\mathrm{C} \subset \boldsymbol{M}$ also exists. A subsidiary cognitive mental system is described in (Adler & Rips 2008, p. 878) that is responsive to the flow of environmental events in the physical world. The states of these subsidiary cognitive systems co-vary with environmental states. The covariance is spatial/temporal with the flow of events in $\mathrm{R}^3(\mathrm{t})$. Physical environmental objects and dynamics are mapped from $\mathrm{R}^3(\mathrm{t})$ to the subsidiary cognitive systems via sensory perception. Sensory perception is a covariant map, $\boldsymbol{g}$, from $\mathrm{R}^3(\mathrm{t})$ to cognitive substructure $\mathrm{C} \subset \boldsymbol{M}$.

In terms of functors, environmental sequences of events $\{A_n\} \subset \mathrm{R}^3(\mathrm{t})$ are co-variantly mapped into sequences of mental representations in the cognitive mind, $\{g(A_n)\} \subset \mathrm{C}$, where $\boldsymbol{g}$ is a covariant functor. Also, mental events $\{E_n\}$ conceived and ordered in the cognitive mind C can be mapped functorialy into sequences of events $\{\boldsymbol{f}(E_n)\}$ in $\mathrm{R}^3(\mathrm{t})$. In this sense, the mental sequence $\{E_n\}$ is a directed class of objects on which a functor (map $\boldsymbol{f}$) is a defined. The map $\boldsymbol{f}$ itself can be defined by physical behavior, or written/verbal commands that carry-out a mental plan in $\mathrm{R}^3(\mathrm{t})$. The map $\boldsymbol{f}$ maps thoughts into behaviors or objects, or events resulting from behaviors.

Typically, a person performing a task (behavior) mentally observes (perceives) the state of what is being produced (mapped into) in $\mathrm{R}^3(\mathrm{t})$, while accomplishing the task- that person is "paying attention" to what he or she is doing. The cognitive mind sums up what it observes in the form of predicative mental representations. The person thinks and produces behavior (task performance) via the map $\boldsymbol{f}: \mathrm{C} \to \mathrm{R}^3(\mathrm{t})$ and simultaneously observes the state of their performance via the perception map $\boldsymbol{g}: \mathrm{R}^3(\mathrm{t}) \to \mathrm{C}$. The typical notion for continuity of a map $\boldsymbol{f}: \mathrm{X} \to \mathrm{Y}$ informs us that $\boldsymbol{f}$ is continuous at $a \in \mathrm{X}$ if "small" changes "near" "a" produce "small" changes "near" $\boldsymbol{f}(a) \in \mathrm{Y}$, and a similar continuity case for map $\boldsymbol{g}: \mathrm{R}^3(\mathrm{t}) \to \mathrm{C}$. In many studies, these changes are spatial.

We are interested in a more general notion of perceptual continuity, with respect to qualitative changes. For instance, if the desire for "salt", $s \in \mathrm{C}$, in the cognitive space increases, there should be some change in a person's physical behavior, $\boldsymbol{f}(s) \in \mathrm{R}^3(\mathrm{t})$, towards increasing salt. Similarly, consider a person perceiving an unripe green banana $b \in \mathrm{R}^3(\mathrm{t})$, via perception map $\boldsymbol{g}: \mathrm{R}^3(\mathrm{t}) \to \mathrm{C}$. As the banana ripens, it qualitative changes from green through yellow to dark brown, in color; from firm to soft, in touch; and from semi-tart to sweet, in taste. These qualitative changes in b should result in cognitive predicative changes in $\boldsymbol{g}(b) \in \mathrm{C}$. These are not spatial changes.

Without the need for metrics defined on the mental space, or the usual Euclidean type metrics on $\mathrm{R}^3(\mathrm{t})$, we define continuity properties for maps relating cognitive and physical spaces, from purely topological definitions.

Definition 3.1

. A function $\boldsymbol{f}\colon \mathrm{C} \to \mathrm{X}$ is called C-continuous if for every clopen set $V \subset X$, $\boldsymbol{f}^{-1}(\mathrm{V})$ is open in C.

Definition 3.2.

Let B be a base for a topology on X. A function $\boldsymbol{f}\colon \mathrm{C} \to \mathrm{X}$ is called B -semi continuous if for every B -semi-open $V \subset X$, $\boldsymbol{f}^{-1}(\mathrm{V})$ is clopen in C. This definition is a specialization of the definition for semi-continuity in (Neeli, 2012, p. 121; Missier & Jesti, 2015, p. 27).

Definition 3.3

. A function $\boldsymbol{f}\colon \mathrm{X} \to \mathrm{Y}$ is continuous if for every open $\mathrm{V} \in \mathrm{Y}$, $\boldsymbol{f}^{-1}(\mathrm{V})$ is open in X.

Example 3.4

. (Cognitive continuity) Define $\boldsymbol{f}\colon \mathrm{C} \to \mathrm{R}^3(\mathrm{t})$ to be a person's behavior (physical, verbal, or written) that maps mental plans in C to events, objects, or entities in $\mathrm{R}^3(\mathrm{t})$. Suppose there is some mental design $H = \{a\ collection\ of\ filters\}$, in an individual's cognitive mind C, for creating or obtaining a physical car K_1, a computer K_2, a dinning room set K_3, a corporation K_4, or a new political party K_5. Here, each cognitive filter is some generalized algebra, $\mathcal{F}_{n_i} = (F_{n_i}, \sigma, I)$ over the formulas, involved with obtaining any K_n in the physical world. The signature σ is a set of generalized "grammar" rules on sequences of formulas, and I is a semantic relation, by which, the person gives meaning to the formula elements and signature. Together, σ and I spell-out how formulas (mental representations) go together and in what sequential order so that a desired result K_n is accomplished. In a more definitive form, the finite collection of mental filters $= \cup_n \{\mathcal{F}_{n_i}\}_{i=1}^{m_n}$, so that $\boldsymbol{f}\colon \mathrm{C} \to \mathrm{R}^3(\mathrm{t})$ is defined by $\boldsymbol{f}\left(\cup_i \mathcal{F}_{n_i}\right) = K_n \subset \mathrm{R}^3(\mathrm{t})$.

$\boldsymbol{f}$ is C-continuous on $\boldsymbol{H}$:

Each $K_n \subset \mathrm{R}^3(\mathrm{t})$ in this example is a complete practical whole and is therefore a clopen set with respect to the "Practical Topology" (Definition 2.1.13). Now, $\boldsymbol{f}$ is C-continuous (Definition 3.1) on H since for any practical whole, clopen K_n, $\boldsymbol{f}^{-1}(K_n) = \cup_i \mathcal{F}_{n_i}$, where each filter $\mathcal{F}_{n_i}$ is open with respect to the cognitive topology (Definition 2.3.4), so that $\boldsymbol{f}^{-1}(K_n)$ is open.

***f* is B -semi continuous on *H*:**

Also, under the "Practical Topology", each practical whole K_n is B -semi-open (Definition 2.1.16 and Proposition 2.1.17). Let H be organized into cognitive ultrafilters (large data sets), $\mathcal{F}_{n_i} \in H$, with Stone base B $= \{\widehat{Y_{n_m}}\}$ (Definition 2.3.5), where

$\widehat{Y_{n_m}} = \{\mathcal{F}_{n_i} \in H | \, Y_{n_m} \in \mathcal{F}_{n_i}\}$ for some subsets Y_{n_m} of cognitive formulas involving a perceived aspect m of K_n. Now, any union of a collection of base elements $\widehat{Y_{n_m}}$ is clopen (Proposition 2.3.6). In this case $\boldsymbol{f}: H \rightarrow \mathrm{R}^3(\mathrm{t})$ is B -semi continuous (Definition 3.2) on H, since for every B -semi-open $K_n \subset \mathrm{R}^3(\mathrm{t})$, $\boldsymbol{f}^{-1}(K_n) = \bigcup_m \widehat{Y_{n_m}}$ is clopen. Cognitively, the inverse, $\boldsymbol{f}^{-1}$, identifies an entity K_n in $\mathrm{R}^3(\mathrm{t})$ to neighborhoods of ultrafilters of perceived aspects of K_n, $\{\widehat{Y_{n_1}}, \widehat{Y_{n_2}}, \ldots, \widehat{Y_{n_p}}\}$ in H.

Example 3.5.

(Cognitive continuity) Define $\boldsymbol{g}$: $\mathrm{R}^3(\mathrm{t}) \rightarrow \mathrm{C}$ to be a person's sensory perception, that maps any perceived physical object, dynamics, events, or entities in $\mathrm{R}^3(\mathrm{t})$ into their cognitive mental subspace C, via the five senses. If a person perceives any entity $K_n \subset \mathrm{R}^3(\mathrm{t})$, we will consider that the entity is mapped, via $\boldsymbol{g}$, to some finite collection of mental cognitive filters $H = \bigcup_n \{\mathcal{F}_{n_i}\}_{i=1}^{m_n}$ in C, where each filter is some mental "knowledge space" associated with an aspect of K_n.

Take the class of filters, $H_R = \bigcup_j \{\mathcal{F}_j\}_{j=1}^{q}$ in the person's mind, formed only by aspects associated with physical entities in $\mathrm{R}^3(\mathrm{t})$. Let B be a base for the Practical topology on some perceptual environment E in $\mathrm{R}^3(\mathrm{t})$. Then, for every filter $\mathcal{F}_j \in H_R$, it contains an aspect(s) associated with some entity or entities $K = \bigcup B_i$ in E, that are some finite union(s) of elements B_i from B. If K is a complete but not yet practical whole, it is open. If K is a practical whole, it is clopen. In any case, we can consider K to be open. Each filter $\mathcal{F}_j$ is an open base element for a cognitive topology (Definition 2.3.4).

***g* is continuous on *E*:**

Let E be an environment in $\mathrm{R}^3(\mathrm{t})$. The sensory perception map $\boldsymbol{g}$: $\mathrm{E} \rightarrow H_R$, defined by the finite union $\boldsymbol{g}(K) = \bigcup_j \mathcal{F}_j$ is continuous, since for any finite union of open filters $\bigcup_j \mathcal{F}_j$ in H_R, there exists $K = \bigcup B_i$ in E, such that $\boldsymbol{g}^{-1}\left(\bigcup_j \mathcal{F}_j\right) = K$, which is open.

There is also the case where a person perceives their own thoughts. Here, there is a "non-sensory" type of perception where a person could think of a "Lion", for instance, and mentally study their own idea of a lion. They are aware (perceive) that they are parsing their mental image of the lion into particular perceived aspects. This mind to mind perception suggests a "non-sensory" perceptual map $\boldsymbol{p}$: $M \rightarrow M$ from the mind to itself. The mind-$\mathrm{R}^3(\mathrm{t})$ relations, maps $\boldsymbol{f}$ and $\boldsymbol{g}$, also suggests that there is some kind of mental covering space for $\mathrm{R}^3(\mathrm{t})$.

4. A COGNITIVE COVERING SPACE FOR $\mathbf{R}^3(\mathbf{t})$

Define the set P to be the mental representations of predicates that a person uses to develop a mental formula ϕ of a "thing". Say a person sees and touches a box. That person could develop mental representations of the box involving descriptors for dimension, color, texture, shape, etc. Each of the descriptors is a member of some distinct predicate set. Subject to the observer, some mental predicates form that can apply to the entire entity, e.g., "the box is blue", "the box is cubic", "the box is heavy". Here, "is blue", "is cubic", and "is heavy" are each associated with the entire box. Similarly, non-physical aspects of a person can be said of the "whole" person, such as, "Kareem is intelligent", "Kareem is discipline", and "Kareem is tenacious". Each predicate applies to Kareem's whole person.

Definition 4.1

. (Covering Space) Let X be a topological space. A covering space of X is a topological space C together with a continuous surjective map $\boldsymbol{f}: C \to \mathrm{X}$, such that {\displaystyle p\colon C\to X\,}for every $x \in \mathrm{X}$, there exists an open neighborhood of x, $U(x)$, where
$\boldsymbol{f}^{-\mathbf{1}}\big(U(x)\big) = \bigcup_j V_j$, all V_n are disjoint open sets in C, and each V_n maps homeomorphically onto U(x) by $\boldsymbol{f}$.

Definition 4.2

. (p-homeomorphic) We say that a mental predicate neighborhood V_p is predicate-homeomorphic (p-homeomorphic) onto an entity K, if it contains a predicate $p \in V_p$ that can be said about the "entire" entity. The mental "act" of identifying mental-predicates with entities defines a map from the predicate space to the space of entities.

Example 4.3

. In the command, "look at the rectangular subway map"; the entire map is categorized as a subway map, and the entire map is rectangular in shape. Thus, the categorical neighborhood for type of map, $V_{\mathbf{s}}$ where s = subway, is p-homeomorphic onto the map = K; and the geometric neighborhood for rectangular, $V_{\mathbf{r}}$ where
r = rectangular, is p-homeomorphic onto map = K. Thus, there is some
p-homeomorphism that maps a mental predicate neighborhood $V_{\mathbf{p}}$ onto K.

Proposition 4.4.

(Cognitive Covering Space) Consider a person's mental map from their subspace P of predicates onto some environment $\mathrm{E} \subset \mathrm{R}^3(\mathrm{t})$ that they operate in, $\boldsymbol{f}: \mathrm{P} \to \mathrm{E}$, where $\boldsymbol{f}$ is cognitive identification of a predicate x with some entity w in E; or, "part" of w. Let the Practical topology (Definition 2.1.13) be the topology on E, generated by base B (Definition 2.1.2). Then P is a covering space for environment E with covering map $\boldsymbol{f}$.

For any perceived entity K in a person's environment E, where $K = \bigcup_{i=1}^{q} B_i$ for some $B_i \in \mathcal{B}$ so that K is open, there exists some cognitive description of K involving a finite set of distinct

mental predicates $\{x_m\}$, with respective disjoint set of predicate neighborhoods $\{V_{x_m}\}$, where $\boldsymbol{f^{-1}}(K) = \cup V_{x_m}$ and each V_{x_m} maps
p-homeomorphically onto K by mental identification $\boldsymbol{f}$. That is, $\boldsymbol{f}$: $V_{x_m} \rightarrow K$ is a
p-homeomorphism, where $\boldsymbol{f}(V_{x_m}) = K$ for each x_m.

Example 4.5.

Consider a person's environment $\mathrm{E} \subset \mathrm{R}^3(\mathrm{t})$ where there are some items, say, $K_1 =$ a box, $K_2 =$ a pair of shoes, $K_3 =$ a computer chip, etc. Also consider that the person belongs to some political party, K_4. The political party is also an entity in E. Each K_m is either complete but not a practical whole (computer chip K_3); an irreducible set that is a practical whole (box K_1); or a disjoint practical whole (pair of shoes K_2, political party K_4). Thus each K_m is open since it is a member of the base B in definition 2.1.2, or some finite union of elements from B. Let the person have a mental description of each entity, in terms of some mental predicates in P. Let $\{x_{m_j}\} \subset P$ be a subset of predicates that the person identifies with some K_m in E, determining the map $\boldsymbol{f}$: $\cup_j \{x_{m_j}\} \rightarrow \mathrm{E}$.

Here are some distinct subsets of mental predicates, P_{m_j}, that the person may use to describe aspects of each entity K_m;

$K_1 = box$:
$\mathrm{P}_{1_1} = \{the\ set\ of\ colors\}$; $\mathrm{P}_{1_2} = \{the\ set\ of\ shapes\}$;
$\mathrm{P}_{1_3} = \{the\ set\ of\ shapes\}$;
$\mathrm{P}_{1_4} = \{the\ set\ of\ texture\ characteristics\}$.

For any $x_{1_j} \in \mathrm{P}_{1_j}$, that can be said of the entire box, there exists a respective open predicate neighborhood $\mathrm{V}_{1_j}(x_{1_j})$ in P_{1_j} such that each $\mathrm{V}_{1_j}(x_{1_j})$ is p-homeomorphic (Definition 4.2) onto the box K_1, under the map $\boldsymbol{f}$, $\boldsymbol{f^{-1}}(K_1) = \cup \mathrm{V}_{1_j}(x_{1_j})$, and $\mathrm{V}_{1_q}\left(x_{1_q}\right) \cap \mathrm{V}_{1_h}(x_{1_h}) = \emptyset$ for $q \neq h$, since all P_{1_j} are disjoint from each other.

$K_2 = pair\ of\ shoes$:
$\mathrm{P}_{2_1} = \{the\ set\ of\ colors\}$; $\mathrm{P}_{2_2} = \{the\ set\ of\ styles\}$;
$\mathrm{P}_{2_3} = \{the\ set\ of\ sizes\}$;
$\mathrm{P}_{2_4} = \{the\ set\ of\ texture\ characteristics\}$.

For any $x_{2_j} \in \mathrm{P}_{2_j}$, that can be said of the entire pair of shoes, there exists a respective open predicate neighborhood $\mathrm{V}_{2_j}(x_{2_j})$ in P_{2_j} such that each $\mathrm{V}_{2_j}(x_{2_j})$ is
p-homeomorphic (Definition 4.2) onto the pair of shoes K_2, under the map $\boldsymbol{f}$, $\boldsymbol{f^{-1}}(K_2) = \cup \mathrm{V}_{2_j}(x_{2_j})$, and $\mathrm{V}_{2_q}\left(x_{2_q}\right) \cap \mathrm{V}_{2_h}(x_{2_h}) = \emptyset$ for $q \neq h$.

$K_3 = computer\ chip$:
Similarly, disjoint open predicate neighborhoods $V_{3_j}(x_{3_j})$ can be associated with the computer chip K_3, such that each $V_{3_j}(x_j)$ is p-homeomorphic onto K_3 under the map $\boldsymbol{f}$,

$$\boldsymbol{f}^{-1}(K_3) = \cup V_{3_j}(x_{3_j}), \text{ and } V_{3_q}\left(x_{3_q}\right) \cap V_{3_h}(x_{3_h}) = \emptyset \text{ for } q \neq h.$$

$K_4 = political\ party$:
$P_{4_1} = \{types\ of\ philosophies\}$; $P_{4_2} = \{finacial\ status\}$;
$P_{4_3} = \{types\ of\ people\}$; $P_{4_4} = \{objectives\ and\ tasks\}$;
$P_{4_5} = \{unity\ status\}$;

For any $x_{4_j} \in P_{4_j}$, that can be said of the entire political party, there exists a respective open predicate neighborhood $V_{4_j}(x_{4_j})$ in P_{2_j} such that each $V_{4_j}(x_{4_j})$ is p-homeomorphic onto the political party K_4 under the map $\boldsymbol{f}$,

$$\boldsymbol{f}^{-1}(K_4) = \cup V_{4_j}(x_{4_j}) \text{ and } V_{4_q}\left(x_{4_q}\right) \cap V_{4_h}(x_{4_h}) = \emptyset \text{ for } q \neq h.$$

In general, the person's mental predicate space P, in this example, is a covering space for their environment E, see Figure 1.

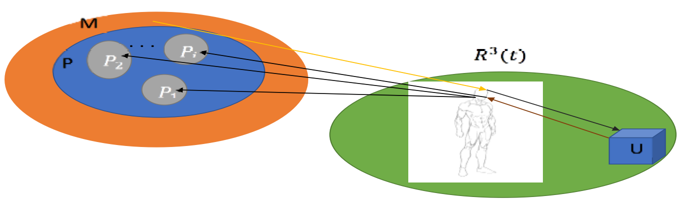

Figure 1 *. The human being in $\boldsymbol{R}^3(\boldsymbol{t})$ is observing a blue box as a neighborhood U, also in $\boldsymbol{R}^3(\boldsymbol{t})$; while simultaneously mapping the physical qualities of the box back to a collection of disjoint sets of predicates representations $\boldsymbol{P_i}$ in the mental predicate covering subspace P of the environment. The covering subspace P is in the mind M.*

5. SEPARATION AXIOMS ON SYSTEMS OF MENTAL FORMULAS

For any individual, the efficiency of problem solving or task performance depends on (1) their mental organization of mental representations (mental formulas ϕ_i) into subsets, (2) the quality of the formulas with respect to solving the problem or task, and (3) which formulas share neighborhoods. There can be many different distinct topologies on a finite spaces of n mental formulas. For instance, the Online Encyclopedia of Integer Sequences indicates that for a set of 4 elements there are 355 different distinct topologies on that set. A set of only 9 elements has 63,260,289,423 distinct topologies. In general, it will be difficult to determine which topology, out of many, that a person will have on their small finite cognitive subsets of knowledge (mental representations). Also, any one person's cognitive topological arrangement of knowledge can change over time.

Certain topologies on finite mental representations (mental data) can be induced by teaching, studying, mental conditioning, or emotional intensity. We can suggest implications for an individual having an indicated topology on a finite cognitive substructure of specified mental data. In the following examples, we note some separation properties induced by a given topology on a cognitive substructure. First some definitions.

Definition 5.1.

(T0 Space) A topological space $\boldsymbol{X}$ is said to be T0 if for every two points $x, y \in \boldsymbol{X}$, there exists an open set $V \subset \boldsymbol{X}$ such that V contains either x or y, but not both.

Definition 5.2.

(R0 Space) A topological space $\boldsymbol{X}$ is said to be R0 if for every two points $x, y \in \boldsymbol{X}$, $x \in \overline{\{y\}}$ if and only if $x \in \overline{\{y\}}$. That is, topologically distinguishable points can be separated.

Definition 5.3.

(T1 Space) A topological space $\boldsymbol{X}$ is said to be T1 if for every two points $x, y \in \boldsymbol{X}$, there exists an open sets $V, U \subset \boldsymbol{X}$ such that V contains x but not y, and U contains y but not x.

Example 5.4.

Consider a training curriculum that prepares executives in business negotiations involving investment and nuclear waste recycling. The curriculum also includes ethics courses. Let b define the set of formulas involving business negotiation, e define the set of ethical rules (formulas), i define the set of knowledge (formulas) involving investment, and n define the set of knowledge (formulas) involving nuclear waste recycling.

Let $C = \{e, b, i, n\}$ be a 4-element cognitive substructure of mental representations of the types of formulas acquired after completing the curriculum. While there are 355 different distinct topologies on C, we will here consider a few;

$\tau_1 = \{\phi, \{e\}, \{e, b\}, \{e, b, i\}, \{e, b, n\}, \{e, b, i, n\}\}$,

$\tau_2 = \{\phi, \{b\}, \{e, b\}, \{e, b, i\}, \{e, b, n\}, \{e, b, i, n\}\}$,

$\tau_3 = \{\phi, \{e, b\}, \{e, b, i\}, \{e, b, n\}, \{e, b, i, n\}\}$,

$\tau_4 = \{\phi, \{e, b\}, \{i, n\}, \{e, b, i, n\}\}$,

$\tau_5 = \{\phi, \{e, n\}, \{b, i\}, \{e, b, i, n\}\}$,

$\tau_6 = \mathcal{P}(C)$, the power set of C.

Cognitive T_0-space:
Topologies τ_1 and τ_2 are T_0. While τ_1 and τ_2 are homeomorphic; semantically, they suggest different conditions on the mindset for persons with the said topologies. For instance, consider persons A_{τ_1} and $B_{\tau_2}$with the same cognitive subspace of mental formulas set C, but topologized under τ_1 and τ_2 , respectively. Under τ_1 , every knowledge-neighborhood contains ethics e; while under τ_2 every knowledge neighborhood contains business b, where business negotiation can also be isolated from ethics. This suggests that person A_{τ_1} is more ethically vigilant than person B_{τ_2}. For A_{τ_1}, every sequence or train of thought converging to b, i, or n, will also converge to also to ethics e. For B_{τ_2} , there is a sequence or train of thought converging to business b uniquely- no ethical input.

Cognitive Indistinguishability and Distinguishability:
For a person, K_{τ_3}, with their cognitive subspace C under topology τ_3 , every neighborhood of business contains ethics and vice-versa. Thus, business and ethics are topologically indistinguishable in thought, while investment and nuclear waste recycling are topologically distinguishable in thought. K_{τ_3} can be said to be more ethically vigilant than B_{τ_2}. The distinguishability between investment and nuclear waste recycling, knowledge-wise may be favorable, since each may be safely discussed, disjoint from each other. The indistinguishability in thought between business and ethics is also favorable since, in general, business actions and negotiation should be tempered by ethics.

Cognitive R_0-space:

Topologies τ_4 and τ_5 are R_0. Every knowledge-neighborhood is clopen. For a person MC , with C topologized under τ_4 , discussions in nuclear waste recycling or investments may not result in business transactions. Since $\{i, n\}$ and $\{e, b\}$ are equivalence classes, no sequence of thought converging to n or i can also converge to b or e for that matter. There is also nothing, topologically, allowing paths (sequences of thought) between $\{i, n\}$ and $\{e, b\}$. As closed sets, $\{i, n\}$ and $\{e, b\}$, sequences originating in $\{i, n\}$ or $\{e, b\}$ will also converge in their set of origination. While business and ethics are topologically indistinguishable in τ_4 ; the isolation and limited cognitive variability of the R_0-space may not contribute much towards actual business with respect to investing in nuclear waste recycling.

Separation by Cognitive Filtration

Cognitive substructures and their properties can also be analyzed in the context of topological Filters. Consider the many-sorted statements (formulas) $\phi_n(\vec{x})$ in a person's cognitive substructure C to be the mental-representations of any perceived or learned unit of knowledge. Now from the power-set of C a collection of Filters $\{F_i\}$ can be constructed from among the elements of $P(C)$. The collection of Filters $\{F_i\}$ whose primitives are sets of statements $\phi_n(\vec{x})$, is a Knowledge-Space. The dynamic cognitive aspect of the collection of filters appears when a person is solving a problem or negotiating a situation; the person, having such an organized collection $\{F_i\}$ in their cognitive mind, can "pervade" the collection, identifying and applying sequences of statements $\{\phi_n\}$ that may or may not lead to solving the problem.

Example 5.5

. (A T1 Topology on subsets of Cognitive Substructure)
Let C be a finite cognitive substructure of mental formulas organized into subsets. Define ultrafilter $\widetilde{\phi_n} = \{A \subset C \mid \phi_n \in A\}$; the set of all subsets of C such that ϕ_n is an element of A, or A is associated with ϕ_n. Let the collection of ultrafilters $\mathfrak{F} = \{\widetilde{\phi_n}\}$ be a base for a topology on the set of subsets of C, the power set of C, $\mathcal{P}(C)$. The power set $\mathcal{P}(C)$ is T1 under the topological base $\mathfrak{F} = \{\widetilde{\phi_n}\}$.

Proof:
For any $A, B \subset \mathcal{P}(C)$ where $A \neq B$, if $\phi_j \in A - B$, then there exists an ultrafilter $\widetilde{\phi_J}$ such that $A \in \widetilde{\phi_J}$ and $B \notin \widetilde{\phi_J}$. Similarly, for $\phi_m \in B - A$, there exists an ultrafilter $\widetilde{\phi_m}$ such that $B \in \widetilde{\phi_m}$ and $A \notin \widetilde{\phi_m}$. The ultrafilters, $\widetilde{\phi_J}$ and $\widetilde{\phi_m}$, are two non-disjoint neighborhoods of A and B respectively. Thus, the power set $\mathcal{P}(C)$ is a T1 space under the topology generated by ultrafilters $\mathfrak{F} = \{\widetilde{\phi_n}\}$.

Let $E \equiv \text{set of statements involving } ethics$ and
$B \equiv \text{ set of statements involving } Business.$
Let $E, B \subset \mathcal{P}(C)$, and let $\mathfrak{F} = \{\widetilde{\phi_n}\}$ be a base for $\mathcal{P}(C)$. Then, there exists non-disjoint ultrafilter neighborhoods $\widetilde{\phi_E} = \{K_i | \phi_E \in K_i\}$ and $\widetilde{\phi_B} = \{P_i | \phi_B \in P_i\}$ of E and B, respectively. While, $B \notin \widetilde{\phi_E}$ and $E \notin \widetilde{\phi_B}$, the intersection of the ultrafilters is not empty, $\widetilde{\phi_E} \cap \widetilde{\phi_B} \neq \emptyset$.

Some results from this T1 space: (1) a cognitive intersection exists, principle filter $\widetilde{\phi_E \phi_B} = \{A_i | \phi_E, \phi_B \in A_i\} \subseteq \widetilde{\phi_E} \cap \widetilde{\phi_B}$ on which to negotiate, that provides a cognitive platform involving business and ethics; and (2) the intersection of these cognitive ultrafilters means that there exists a sequence of subsets (thoughts) connecting E and B.

Definition 5.6

. (Hausdorff Space) A topological space $\boldsymbol{X}$ is said to be Hausdorff (T2) if for every two points $x, y \in \boldsymbol{X}$ there exists open sets $U, V \in \boldsymbol{X}$ such that $x \in U \; and \; y \in V$, and $U \cap V = \emptyset$.

Example 5.7

. (A Hausdorff Cognitive Substructure)
Define $\hat{Y} = \{\widetilde{\phi_n} \in S(C) | \, Y \in \widetilde{\phi_n}\}$, where Y is a subset of formulas in C and $S(C)$ is the space of ultrafilters on C. We think of the ultrafilters $\widetilde{\phi_n}$ as "large" associations of mental data that are taken as units of information in the mind.

Let $\mathrm{B} = \{\hat{Y}\}$ be a Stone base for $S(C)$, whose "points" (units) are now ultrafilters. Under the topology generated by the Stone base, $S(C)$ is now a totally disconnected Hausdorff space, where every point can be isolated. That is, there exists single point open neighborhoods containing one ultrafilter. In particular, the ultrafilters
$\widetilde{\phi_E} = \{K_i | \phi_E \in K_i\}$ and $\widetilde{\phi_B} = \{P_i | \phi_B \in P_i\}$ involving ethics and business, respectively, can be completely separated from each other by their disjoint Stone neighborhoods $\widehat{Y_{\widetilde{\phi_E}}}$ and $\widehat{Y_{\widetilde{\phi_B}}}$.

For the individual under this "Stone" cognitive ultrafiltration, the entire mental formula units of ethics $\widetilde{\phi_E}$ and business $\widetilde{\phi_B}$ can be separated and distinct in that person's cognitive mind, during business negotiations. All convergent sequences of thought are unique. Those converging to business will not also converge to ethics, and visa-versa. This suggest the possibility that the individual can negotiate without an ethical mindfulness.

Emotion and Hausdorff-ness Revisited

Another interesting observation is the relationship between cognitive mental substructures C and the perceptual physical world $R^3(t)$, where emotional or physical trauma can impose Non-Hausdorff properties on C, with respect to physical behavior expressing structural contents of C.

Example 5.8

. (An induced Cognitive Equivalence Class). Consider a case where a war veteran was exposed to the traumatic event $\mathbf{E} \subset R^3(t)$ of an explosion due to a cylindrical can-like bomb q. Further, the veteran had to execute a particular behavior, $\mathbf{B} \subset R^3(t)$, to survive. Based on that individual's mental condition and personality, the emotional trauma could act on a cognitive substructure of their mind to "solidify" all mental representations associated with the traumatic event. Any thoughts, images, smells, sounds, or even tastes associated with the traumatic event can cause the person to unconsciously "relive" the event, which is a case of post-traumatic stress syndrome (Foa, Steketee, & Rothbaum, 1989, p. 155).

Let $\boldsymbol{g}: \mathrm{R}^3(\mathrm{t}) \to \mathbf{C}$ be the Veteran's continuous sensual/perceptual map from $R^3(t)$ to their cognitive substructure $\mathbf{C}$. Now $\mathcal{E} = \{x_i \mid x_i = \boldsymbol{g}(r_i)\} \subset \boldsymbol{C}$ is the set mental images of the perceived elements of the traumatic event $E = \{r_1, r_2, \dots, r_n\} \subset R^3(t)$, where the cylindrical can-like bomb $q \in E$. If due to high emotional stress, the mental images (elements) $x_i = \boldsymbol{g}(r_i)$ of the traumatic event "solidify" in $\mathbf{C}$, then the Veteran identifies every element in $\mathcal{E}$ with any other element in $\mathcal{E}$. Thus the elements of $\mathcal{E}$ are in some equivalence relation, $\sim$, with the image of the cylindrical bomb q. $\mathcal{E}$ becomes the equivalence class $\mathcal{E} = \{x_i | x_i \sim \boldsymbol{g}(q)\} = [\boldsymbol{g}(q)] \subset \boldsymbol{C}$. Now, $\boldsymbol{C}$ is a non-Hausdorff cognitive substructure, since every $x_i \neq \boldsymbol{g}(q)$ in $\mathcal{E}$ is in every neighborhood of $\boldsymbol{g}(q)$.
$\mathcal{E}$ is a set of topologically indistinguishable elements in that Veteran's mind. $\mathcal{E}$ is an emotion induced equivalence class. At best, the cognitive subspace $\boldsymbol{C}$ is R0.

Suppose that several years later $\mathcal{E} \subset \boldsymbol{C}$ remains a cognitive equivalence class in the Veteran's mind, and the Veteran is in a supermarket. Let $\{a_n\} \subset \boldsymbol{C}$ be any mental representation of a sequence of events in the supermarket that converges to a cylindrical can falling to the floor in a supermarket- the can resembling the bomb q. Now $\lim_n a_n = \boldsymbol{g}(q)$, where $\mathcal{E} = [\boldsymbol{g}(q)]$ so that the limit converges to every element in $\mathcal{E}$, i.e., $\lim_n a_n = x_i$ for all $x_i \in \mathcal{E}$, which can trigger an associated survival behavior
$\boldsymbol{B} = \boldsymbol{f}(\mathcal{E})$ in the supermarket, where $\boldsymbol{f}\colon \mathbf{C} \to \mathrm{R}^3(\mathrm{t})$ is a functorial behavior map, mapping cognitive thought patterns into behavior in $\mathrm{R}^3(\mathrm{t})$.

DISCUSSION

In this research, we explained some cognitive structures and dynamics of psychological phenomena in purely topological terms. Our topological treatment gave a qualitative mathematical analysis on mental substructures, in the absence of quasi-metrics, pseudo-metrics, or metrics on either the physical world or the mental space. A topological base was defined for cognitive substructures and for physical world environments, from human consideration; that is, humans topologize their physical environment and their own cognitive space. The assignment of topologies on a set is subjective. From covariance between cognitive states and perceived changes in the physical environment, we topologically defined continuity properties for perception and behavior maps between the physical world and cognitive substructures. Cognitive covering spaces were also defined for physical world environments, based on the ability of the human mind to perceive and ascribe properties, qualities, and aspects in the form of predicates. Motivated by the minds ability to analyze and separate sets of experiences, we also explored implications of separation axioms on cognitive substructures, with respect to semantic associations of mental data. We also topologically described the minds ability to "solidify" certain of its contents into equivalence classes, suggested by a Post-Traumatic Stress symptom. We conjecture that there is an inverse relation between emotional intensity and the "Hausdorff-ness" of cognitive substructures.

While this work considers the existence of cognitive substructures $\mathcal{C} = (C, \sigma, I)$ that have a generalized signature σ and interpretation function I on a domain of mental representations C, we do not exploit the theory of interpretations or signature theory in detailed fashion. We were able to explore topological scenarios for cognitive substructures, physical environments, and perceptual maps. However, for any given person, there can be billions of distinct topologies on finite sets of cognitive data. Which topology will they choose? The answer to this question involves a semantic problem on which interpretation theory bears.

How the mind collects its mental representations into meaningful subsets of "associated" data that support daily activities, or even the mental act of understanding, poses a problem involving data semantics and brings into question the part of the cognitive structure involving interpretation- the interpretation function. Is the mind "doing" the interpretation, via itself, or is the "person" doing the interpretation via the mind? Whichever the case may be, the collection of mental data into meaningful associations requires interpretation. Formally, the interpretation function is an assignment of meaning to a domain of symbols and even assigns meaning to the signature on that domain. Take the mental representations of "things" in some subset of $R^3(t)$. Those mental representations serve as a metalanguage C from which to define and negotiate the object-language (objects and events) of environments E in $R^3(t)$. If E is interpreted in some cognitive metalanguage C, then the model theory defines the interpretation as an onto n-nary map $(n, f): C^n \rightarrow E$ such that for all objects or events $K \subseteq E^m$ definable in E^m, the preimage of K, $(m, f^{-1})(K) \subset C$, is definable in C. In this research, the formation and dynamics of cognitive topologies can be investigated with respect to given "definability" rules, degrees of an interpretation, or under different interpretations such as isomorphisms or retractions (Visser, 2017, p. 284). An interesting case to study is interpretations for classes of formulas $\{\phi_n\}$ in cognitive metalanguage C having no definable images in $R^3(t)$, such as pure imagination or ideas yet to become physical inventions. Under the theory of interpretations, we can also explore emotion as a derivative of interpretation, and emotional influence on cognitive topologies and human behavior.

Mental representations (data) and their organization in the cognitive mind also depends on the "mental grammar"- a set of mental laws or rules by which mental data is combined or concatenated. Typical concatenation rules in logic to model reasoning involve the binary connectives "and" $(\wedge)$, "or" $(\vee)$, and "implication" $(\longrightarrow)$; and in set theory, the union and intersection. The various sorts and modes of mental data suggests that there can be various n-nary, n-sorted functions along with syntactic rules in the signature σ for any one person's cognitive metalanguage C (Fodor & Pylyshyn, 1988, p. 3). For instance, what would be the n-nary function combining the many sorted mental data for a person at the zoo writing notes on an elephant, while chewing gum, observing the elephant, and hearing and smelling the elephant? The n-nary function and syntactic rules would require the mental processing (combining) of geometric, human language, taste, odor, and sound data into a formula ϕ "satisfiable" in C. Here, satisfiable means that the formula makes sense to the person.

Additional research in the Psychological sciences on mental grammars (syntactic laws or rules) on cognitive spaces could extend this research under a more algebraic topological approach. Also, while (Phillips & Wilson, 2010) explains cognitive systematicity via the universal property of free cognitive structures and adjunctions in a category theory, independent of grammars or topological bases, our research can be integrated into cognitive studies implementing category theory. Quantum Cognition is another focus in mathematical psychology involving the application of quantum mechanics to explain the cognitive contextual interaction resulting from the superposition of perceived ideas during decision making (Aerts, 2009, p. 314). Our research in topological psychology can also be applied to describe the dynamics of cognitive substructures involved with quantum Cognition.

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